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Time in a Simple Model of a Physical System

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Abstract

We build a model of time starting from the primitive concept of base-set $B \equiv \{\alpha_i | i \in I\}$ of all physical systems, whose elements are called pre-particles α_i. We assume that B is a denumerably infinite set. Particles or bodies are represented by the subsets of the power set $\mathscr{P}(B)$ of the base-set B. A physical system is represented by a set of particles. We introduce the distinction between evolving and non-evolving particles, and assume that the former are represented by those subsets of $\mathscr{P}(B)$ which are chains. Making use of the above concepts we define the state of a particle and the indicator of the state of a particle with respect to a given state of the same or another particle. Then we define in terms of indicators the concepts of instant, time-set, degenerate time-set, event, and clock. For the time related to a given clock one has a set in which the order relation is in general not connected. Some theorems are proved.

1. *Introduction*

There are several fundamental theories of time. Among them we can mention the theory of Noll and Bunge (Noll, 1967; Bunge, 1967, 1968, 1970), and the theory of Basri (1966). The first starts from the primitive concept of an event and a function that pairs couples of events to real numbers This function produces a partition into equivalence classes of the set of events and orders the corresponding quotient set. Each of these equivalence classes is defined as an instant. In the paper of Noll (1967) a theory of universal time (classical) is developed. Bunge (1967. 1968, 1970) improves the theory by introducing new concepts, among others, that of reference frame, allowing the definition of local time, which fits in with the relativistic schema.

More recently, Bunge (1974) has developed a theory of time starting from the primitive concepts of a *thing* and of the *property* of a thing. Then, the concept of state space of a given thing is defined in terms of the primitive concept of property. The state space for several non-interacting things is given by the cartesian product of the individual state spaces. Some of the states belonging to a given state space can be ordered by general laws. Given a state space, the concepts of event and instant are defined in the subspace of ordered states as ordered pairs of states, and equivalence classes of states, respectively. One of the properties in this theory is that the disappearance of things entails the non-existence of time.

Basri's theory starts from the primitive concepts of observer, sensations, subjective entities, objective entities, appearance and disappearance events, and other primitive concepts (Basri, 1966). Fixing some parameters this theory yields a model which has some of the properties of general relativity.

In the present paper we develop a model of time starting from the primitive concept of base-set B of all physical systems. $B \equiv \{\alpha_i \mid i \in I\}$. is a denumerably infinite set of what we call pre-particles α_i. The concept of pre-particle is a primitive one, and for the sake of intuition we will say that a pre-particle is a physical object which can be described only from the outside, i.e., there is not a description of its internal properties.

Let us consider the power set $\mathscr{P}(B)$ of the set B, i.e., the set whose elements are all the subsets of B, given by $\mathscr{P}(B) \equiv \{a(x) \mid x \in \tilde{R} \subseteq R\}$, where R is the set of the real numbers. Because B can be either finite or denumerably infinite, $\mathscr{P}(B)$ will be either finite or infinite with the power of the continuum, respectively (Fraenkel, 1958). A particle or body is represented by any subset

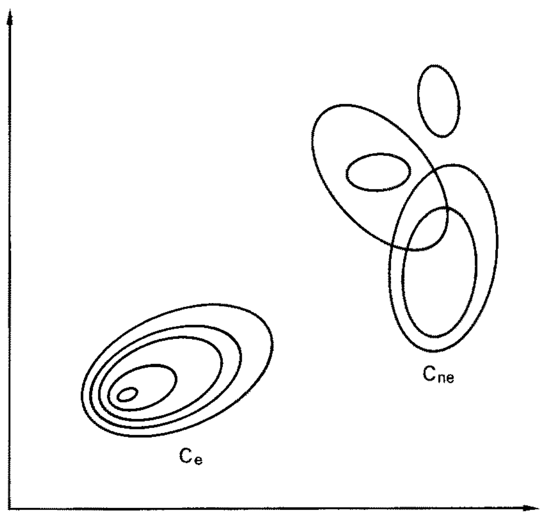

Figure 1.—A picture of an evolving particle (left) and of a non-evolving particle (right). In order to have an infinitely denumerable base-set B of pre-particles, let us consider only the rational points of each coordinate axis. Then, each point whose coordinates are rational numbers represents a pre-particle. In the above convention, c_e represents an evolving particle whose elements are the ovals which cover each other. On the other hand, c_{ne} represents a non-evolving particle because some of its elements are not ordered by the proper inclusion relation $\subset$.

of $\mathscr{P}(B) - \emptyset$, where $\emptyset$ is the empty set, and we define a physical system as a set of particles. According to the above definitions there are particles which are represented by sets of only one set which in turn contains only one pre-particle.

Consider the physical system $\mathbb{C}\mathscr{P}(B)$ whole elements are the subsets of $\mathscr{P}(B)$ which are chains (Fraenkel & Bar-Hillel, 1958). A chain is a set $\{d(x) \mid x \in X\}$ ordered by the relation $\subset$ of proper inclusion, such that for any $x, x' \in X$ one has either $d(x) \subset d(x')$ or $d(x') \subset d(x)$. As is well known the relation $\subset$ can be defined in terms only of the membership relation $\in$, and, in general, any order relation can be defined in terms of the membership relation (Fraenkel & Bar-Hillel, 1958). This implies that in our model the basic relation is that of membership.

We distinguish between two types of particles: if $c \in \mathbb{C}\mathscr{P}(B)$ we say that c represents an evolving particle, and if $c \in \mathbb{C}\mathscr{P}(B)$ then c represents a non-evolving one (Fig. 1). To give an example, let us suppose that the base-set of pre-particles is finite and given by $B \equiv \{\alpha_1, \alpha_2\}$. Then, $\mathscr{P}(B) \equiv \{\emptyset, \{\alpha_1\}, \{\alpha_2\}, \{\alpha_1, \alpha_2\}\}$, and the particles are represented by the following subsets of the set $\mathscr{P}(B) - \emptyset$: $c_1 = \{\{\alpha_1\}\}, c_2 \equiv \{\{\alpha_2\}\}, c_3 \equiv \{\{\alpha_1, \alpha_2\}\}$, $c_4 \equiv \{\{\alpha_1\}, \{\alpha_1, \alpha_2\}\}, c_5 \equiv \{\{\alpha_2\}, \{\alpha_1, \alpha_2\}\}$, and $c_6 \equiv \{\{\alpha_1\}, \{\alpha_2\}, \{\alpha_1, \alpha_2\}\}$. A physical system will be represented by a set of c_i's. $\mathbb{C}\mathscr{P}(B)$ consists of $\{c_1, c_2, c_3, c_4, c_5\}$; and c_6 represents a non-evolving particle. (From now on, we will speak about the c's as particles; but it is understood that the c's are representations of particles, in the sense that the c's are sets, i.e., concepts, whereas the particles are things (Bunge, 1974)).

Given a particle $c_i \equiv \{a^i(x) \mid x \in X \subseteq R\}$, where by definition $a^i(x) \in \mathscr{P}(B)$, we call each $a^i(x)$ an *element* of c_i. Further, we call a *state* of c_i any set

$$s^i(x) \equiv a^i(x) - \bigcup_{x' \in X'(x)} a^i(x')$$

where the $a^i(x')$ with $x' \in X'(x)$ are all the elements of c_i such that $a^i(x) \neq a^i(x')$ and $a^i(x) \not\subset a^i(x')$. Therefore, if all the elements of c_i are disjoint, each $a^i(x)$ is in turn a state of c_i. On the other hand, if c_i is an evolving particle with more than one element, some states of c_i can be different from any $a^i(x)$ (Fig. 2). In the above example, c_1 has the state $\{\alpha_1\}$, c_2 the state $\{\alpha_2\}$, c_3 the state $\{\alpha_1, \alpha_2\}$. c_4 the states $\{\alpha_1\}$ and $\{\alpha_1, \alpha_2\} - \{\alpha_1\} = \{\alpha_2\}$, c_5 the states $\{\alpha_2\}$ and $\{\alpha_1, \alpha_2\} - \{\alpha_2\} = \{\alpha_1\}$, and c_6 has the states $\{\alpha_1\}, \{\alpha_2\}$ and $\{\alpha_1, \alpha_2\} - \{\alpha_1, \alpha_2\} = \phi$. We see that different particles can have the same states, but they differ in the order of the states. This order is characterized by the following definition. The states of an evolving particle can be ordered in the following way: If $c_m \equiv \{a^m(x) \mid x \in X \subseteq R\}$ is an evolving particle, then two states

$$s^m(x) = a^m(x) - \bigcup_{x' \in X'(x)} a^m(x') \qquad \text{and} \qquad s^m(y) = a^m(y) - \bigcup_{y' \in X'(y)} a^m(y')$$

where $x, y \in X$, are such that $s^m(x) < s^m(y)$ iff $a^m(x) \subset a^m(y)$. For a non-evolving particle the order thus induced in the set of states of a particle, will not be connected. In the above example, the states of c_4 are such that

$\{\alpha_1\} < \{\alpha_2\}$, and for the states of c_5 we have $\{\alpha_2\} < \{\alpha_1\}$. On the other hand, for c_6 we have neither $\{\alpha_1\} < \{\alpha_2\}$ nor $\{\alpha_2\} < \{\alpha_1\}$.

We give now an intuitive representation of some of the above concepts. A pre-particle can be visualized as a physical object which has no parts and does not change. For a particle $c_i = \{a^i(x) \,|\, x \in X \subseteq R\}$, the cardinal of the set $\bigcup_{x \in X} a^i(x)$ gives a measure of the spatial region which c_i sweeps in its movement. An evolving particle occupies at different stages of its evolution a spatial region whose measure is given by the cardinal of the corresponding state.

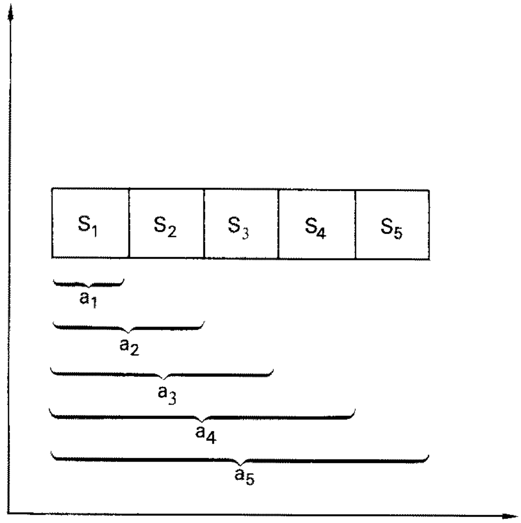

Figure 2.—States of an evolving particle. The elements of c are $a_1 = s_1$, $a_2 \equiv s_1 \cup s_2$, $a_3 \equiv s_1 \cup s_2 \cup s_3$, $a_4 \equiv s_1 \cup s_2 \cup s_3 \cup s_4$ and $a_5 \equiv s_1 \cup s_2 \cup s_3 \cup s_4 \cup s_5$. The states of c are given by s_1, s_2, s_3, s_4 and s_5. The elements of c are ordered by the relation $\subset$ as $a_1 < a_2 < a_3 < a_4 \lesssim a_5$. According to the rule given in the text, the states of c are ordered as $s_1 < s_2 < s_3 < s_4 \lesssim s_5$.

It is important to point out that the introduction of the notion of an evolving particle does not imply the surreptitious introduction of the notion of time. What we are introducing here is something related to the concept of change, which is different from time (for a discussion of the difference between change and time see for instance Bunge (1968)). But one may then wonder if we are in fact working with absolute movement. However, because the measurement of the rate of a movement requires clocks and reference frames and, as we will see, in our model measurement of time intervals with a given clock produces a partition of the set of instants that depends on the clock, this model provides a flexible formalism for the measurement of time intervals. At any rate, in the present stage of the model we say nothing about separations in space between particles, and between different states of a particle. Thus, strictly speaking, we cannot describe either movements or reference frames consisting in more than one clock.

2. *The Model*

In what follows we make implicit use of the postulates of the Zermelo-Fraenkel version of set theory (Fraenkel & Bar-Hillel, 1958).

Postulate 1. There exists a denumerably infinite set $B \equiv \{\alpha_i \mid i \in I\}$ which we call the base-set of pre-particles α_i.

Postulate 2. Any subset of the power set $\mathscr{P}(B)$ of B represents a particle of the physical world.

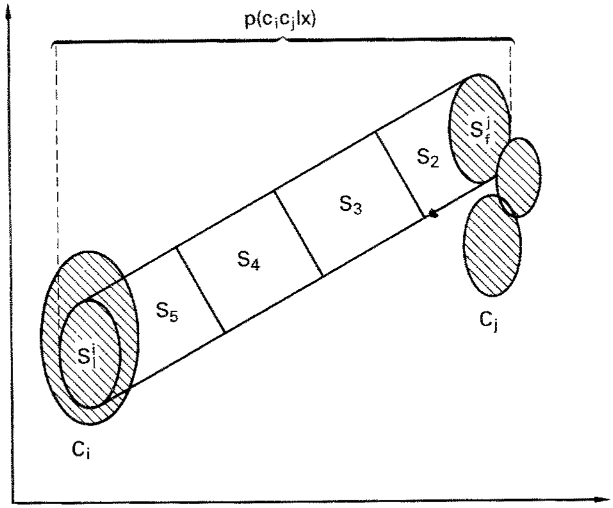

Figure 3.—Indicator of a state. The two sets of hatched ovals represent the paricles c_i and c_j. c_i is an evolving particle and c_j is a non-evolving one. s_f^j is a state of c_j and in this particular case is also an element of the same particle. s_f^j is also the first element and the first state of the indicator $p(c_i c_j \mid x)$. s_l^i is a state of the particle c_i and is also the last state of the indicator $p(c_i c_j \mid x)$. The last element of $p(c_i c_j \mid x)$ is $s_f^j \cup s_2 \cup s_3 \cup s_4 \cup s_5 \cup s_l^i$.

Postulate 3. There exist two types of particles in the physical world, namely, the evolving particles, which are represented by those subsets of $\mathscr{P}(B)$ that are chains, and the non-evolving particles, represented by those subsets of $\mathscr{P}(B)$ that are not chains.

Definition 1. We call an *indicator* of a state $s^j(x)$ of c_j with respect to the state $s^i(y)$ of c_i, any evolving particle with a first and a last state which are equal to $s^j(x)$ and $s^i(x)$, respectively (Fig. 3).

According to this definition an evolving particle c_i with a first and a last state which are denoted by s_1 and s_2, respectively, is an indicator of the state s_1 of c_i with respect to the state s_2 of the same particle c_i. Also, for an evolving particle c_j whose first and last states are s_1 and s_2, respectively, and for a particle c_i with one of its states equal to s_2, it follows then that c_j is an indicator of the state s_1 of c_j with respect to the state s_2 of c_i. The case of an indicator of a state of a given particle with respect to a state of another particle, is illustrated in the Fig. 3.

Theorem 1. For any state $s^i(x)$ of a given c_i, there exists exactly one indicator $p(c_ic_i|z_0)$ of the state $s^i(x)$ of c_i with respect to the same state $s^i(x)$ of c_i, and $p(c_ic_i|z_0)$ has only one element which is equal to its first and last element.

Proof. First of all it is important to realize that an evolving particle c_j with more than one element necessarily has more than one state. Let $a^j(x_0)$ be the first element of c_j. Then, $a^j(x_0)$ is also the first state of c_j, because this state must be such that

$$s^j(x_0) = a^j(x_0) - \bigcup_{x' \in X'(x_0)} a^j(x')$$

with $a^j(x') \in c_j$, $a^j(x') \neq a^j(x_0)$ and $a^j(x_0) \not\subset a^j(x')$; however $a^j(x_0)$ is such that for any $a^j(x') \in c_j$ one has $a^j(x_0) \subset a^j(x')$, i.e.

$$\bigcup_{x' \in X'(x)} a^j(x') = \emptyset$$

On the other hand, if more than one element belongs to c_j one has $a^j(x_0) \subset a^j(x')$, i.e.

$$s^j(x') = a^j(x') - \bigcup_{x'' \in X''(x')} a^j(x'')$$

and $a^j(x_0) \subset \cup a^j(x'')$ and $\cup a^j(x'') \subset a^j(x')$, for any $a^j(x') \in c_j$ different from $a^j(x_0)$. Therefore, $s^j(x') \neq a^j(x_0) = s^j(x_0)$. Now, according to the hypothesis, the indicator $p(c_ic_i|z_0)$ must be such that its first state be equal to its last one, and thus $p(c_ic_i|z_0)$ must have only one state which is $s^i(x_0)$. Therefore, taking into account the above argument, only one element which is equal to the first and last state of $p(c_ic_i|z_0)$, belongs to $p(c_ic_i|z_0)$, and thus $p(c_ic_i|z_0)$, is unique.

In general, there is more than one indicator between two given particles. We will use the notation $P(c_ic_j)$ for the set to which all the indicators of the states of c_j with respect to the states of c_i belong, and the notation $p(c_ic_j|x)$ for the indicators belonging to $P(c_ic_j)$, i.e. $P(c_ic_j) \equiv \{p(c_ic_j|z) | z \in Z \subseteq R\}$, where z is an index which distinguishes the elements of $P(c_ic_j)$. On the other hand, given a physical system S we use the notation $PS(c_ic_j)$ for the set $S \cap P(c_ic_j)$.

Definition 2. Given a physical system S and a particle $c_i \subset \mathscr{P}(B)$, an instant of S with respect to c_i is defined as the set $t(c_i; S) \equiv \{p(c_ic_j|z_j) | j \in J$, provided that $p(c_ic_j|z_j) \in PS(c_ic_j)$, and that the last state $s^j(x_l)$ of $p(c_ic_j|z_j)$ be the same state of c_i for all $j \in J$, and that to the intersection $PS(c_ic_j) \cap t(c_i; S)$ there belong no more than one indicator for every $j \in J\}$.

Definition 3. We define the set $T(c_i; S)$ containing all the $t(c_i; S)$ as the time-set of S with respect to c_i.

Definition 4. An event of c_i with respect to S is a set $e(c_i; S) \equiv \{p(c_j c_i | z_j) | j \in J$, provided that $p(c_j c_i | z_j) \in PS(c_j c_i)$, and that the first state of $p(c_j c_i | z_j)$ be the same state of c_i for all $j \in J$, and that to the intersection $PS(c_j c_i) \cap e(c_i; S)$ there belong no more than one indicator for every $j \in J\}$.

To support the distinction above introduced between instant and event let us make the following intuitive representation: an indicator $p(c_i c_j | x)$ correponds to a signal going from c_j to c_i. Then, an instant $t(c_i; S)$ may be interpreted as the set of all the signals which are evolving particles belonging to S, and with a first and a last element, coming from the particles belonging to S and arriving at the same state of c_i. In the same way, an event $e(c_i; S)$ will be the set of all the signals which are evolving particles belonging to S, and with a first and a last element, coming from the same state of c_i which reach the particles belonging to S.

Theorem 2. Given S and $c_i \subset \mathscr{P}(B)$, for all $p \in PS(c_i c_k)$, where $c_k \in S$, there is at least one instant $t \in T(c_i; S)$ such that $p \in t$.

Proof. Consider $p(c_i c_k | z_k) \in PS(c_i c_k)$, where $c_k \in S$. Because $p(c_i c_k | z_k)$ is an indicator it follows that $p(c_i c_k | z_k)$ is an evolving particle with a first and a last element, and hence with a first and a last state, which we denote by s_1 and s_2, respectively. On the other hand, there exists a state of c_i identical to s_2. Therefore, $p(c_i c_k | z_k)$ fulfils all the conditions for belonging to an instant of $T(c_i; S)$, and thus there exists at least one $t(c_i; S)$ such that $p(c_i c_k | z_k) \in t(c_i; S)$.

Theorem 3. $t, t' \in T(c_i; S)$ and $t \neq t' \Rightarrow t \not\subset t'$ and $t' \not\subset t$.

Proof. Let us suppose $t \subset t'$, i.e. $t' - t \neq \emptyset$, and let $p(c_i c_k | z_k) \in t' - t$. From Definition 2 one has that all the last states of each $p(c_i c_j | z_j) \in t'$ are equal between them, and equal to a given state of c_i, which we may call s_2. Moreover, because $t \subset t'$, all elements of t also belong to t' and thus fulfil the same above condition, i.e. the last state of each indicator belongs t is s_2. Therefore $p(c_i c_k | z_k) \in t$, which contradicts $p(c_i c_k | z_k) \in t' - t$.

Corollary 1. $t, t' \in T(c_i; S)$ and $t \neq t' \Rightarrow t \cap t' \subset t$ and $t \cap t' \subset t'$.

Definition 5. A time-set $T(c_i; S)$ is *degenerate* iff for any $t \in T(c_i; S)$ to which there belongs an indicator of each non-empty $PS(c_i c_k)$, where $c_k \in S_0 \subseteq S$ one has that for all sets $\tilde{t}$'s of indicators defined such that an indicator of each non-empty $PS(c_i c_k)$, where $c_k \in S_0$ belongs to $\tilde{t}$, then $\tilde{t}$ also belongs to $T(c_i; S)$.

Theorem 4. Consider a physical system S and a particle $c_i \subset \mathscr{P}(B)$ such that : (i) $T(c_i; S)$ is a degenerate time set; (ii) for all the $c_j \in S$ one has that $PS(c_i c_j)$ is either finite with at least two elements, or denumerably infinite; (iii) at least one instant which belongs to $T(c_i; S)$ is a denumerably infinite set of indicators. Then, the conditions (i), (ii) and (iii) imply that there exists a set $T_0(c_i; S) \subseteq T(c_i; S)$ such that each of its elements is an infinite denumerable set of indicators, and that the cardinal of $T_0(c_i; S)$ is larger than the cardinal of any $PS(c_i c_k)$, where $c_k \in S$, namely, $T_0(c_i; S)$ has the power of the continuum

Proof. From (iii) in Theorem 4, there exists at least one infinite denumerable set t_1 of indicators such that $t_1 \in T(c_i; S)$. Let us write t_1 as $\{p(c_ic_j|x), p(c_ic_k|y), p(c_ic_l|z), \ldots, p(c_ic_j|v), p(c_ic_t|w), \ldots\}$. Because $T(c_i; S)$ is a degenerate time-set we have that the instant t_2 obtained from t_1 by making the substitution of $p(c_ic_j|x) \in t_1$ by $p(c_ic_j|x') \in PS(c_ic_j)$ with $x' \neq x$, also belongs to $T(c_i; S)$. The existence of $p(c_ic_j|x')$ is guaranteed by the condition (ii) in the above theorem. By repeating the same procedure on t_2 and so on, one establishes the following one-to-one correspondence:

$$1 \leftrightarrow t_1 = \{p(c_ic_j|x), p(c_ic_k|y), p(c_ic_l|z), \ldots, p(c_ic_s|v), p(c_ic_t|w), \ldots\}$$
$$2 \leftrightarrow t_2 = \{p(c_ic_j|x'), p(c_ic_k|y), p(c_ic_l|z), \ldots, p(c_ic_s|v), p(c_ic_t|w), \ldots\}$$
$$3 \leftrightarrow t_3 = \{p(c_ic_j|x'), p(c_ic_k|y'), p(c_ic_l|z), \ldots, p(c_ic_s|v), p(c_ic_t|w), \ldots\}$$
$$\vdots$$
$$m \leftrightarrow t_m = \{p(c_ic_j|x'), p(c_ic_k|y'), p(c_ic_l|z'), \ldots, p(c_ic_s|v'), p(c_ic_t|w), \ldots\}$$
$$\vdots$$

Let A be the set of all the above instants, i.e. $A \equiv \{t_i | i \in N\}$ where N is the set of natural numbers. Consider now the set $T_0(c_i; S)$ to which belong all the elements of $T(c_i; S)$ which are denumerably infinite sets of indicators. Then, by definition, one has $T_0(c_i; S) \subseteq T(c_i; S)$ and also $T_0(c_i; S) \neq \emptyset$ because $A \neq \emptyset$. In order to see that A is a proper subset of $T_0(c_i; S)$ it is sufficient to recognize that $t_i \in A \Rightarrow t_i \in T_0(c_i; S)$ and that the set of indicators $d \equiv \{p_1, p_2, \ldots, p_m, \ldots\}$ constructed following Cantor's diagonal method (i.e. the elements of d are such that $p_1 \neq p(c_ic_j|x), p_2 \neq p(c_ic_k|y), \ldots, p_m \neq p(c_ic_t|w), \ldots$) is such that $d \in T_0(c_i; S)$ and $d \notin A$ (see, e.g. Fraenkel, 1958). In the same way, it can be seen that any denumerably infinite set $\tilde{A}$ of instants which are denumerably infinite set of indicators is a proper subset of $T_0(c_i; S)$, i.e. $\tilde{A} \subset T_0(c_i; S)$. Then, $T_0(c_i; S)$ is a non-denumerably infinite set. Moreover, because $PS(c_ic_m)$ is either finite or denumerably infinite for all $c_m \in S$, one has that the cardinal of $T_0(c_i; S)$ is larger than the cardinal of $PS(c_ic_m)$ for all $c_m \in S$.

In order to see that $T_0(c_i; S)$ has the cardinal $\mathcal{N}_1$ of the set of the real numbers, let $T_{0\alpha}$ be the set $T_0(c_i; S)$ when all the $PS(c_ic_m)$ contain only two indicators, and $T_{0\beta}$ be the set $T_0(c_i; S)$ when all the $PS(c_ic_m)$ are denumerably infinite sets. Then we have $\mathcal{N}(T_{0\alpha}) \leqslant \mathcal{N}(T_0(c_i; S)) \leqslant \mathcal{N}(T_{0\beta})$, where $\mathcal{N}(T)$ stands for the cardinal of the set T. On the other hand, the cardinal of $T_{0\alpha}$ is $\mathcal{N}_1$, because each element of $T_{0\alpha}$ can be written in the form of a continuous fraction in binary system by assigning numbers 0 and 1 to the two indicators belonging to $PS(c_ic_m)$. This establishes a one-to-one correspondence between the real numbers in the interval [0, 1) and the elements of $T_{0\alpha}$. Moreover, the cardinal of $T_{0\beta}$ is $\mathcal{N}_0^{\mathcal{N}_0} = \mathcal{N}_1$, where $\mathcal{N}_0$ stands for the cardinal of a denumerably infinite set. Therefore, $\mathcal{N}_1 \leqslant \mathcal{N}(T_0(c_i; S)) \leqslant \mathcal{N}_1$, i.e. $\mathcal{N}(T_0(c_i; S)) = \mathcal{N}_1$.

Let us now consider a time-set $T(c_i; S)$, an evolving particle $c_j \in S$, and the set $Z(c_ic_j;S)$ such that $Z(c_ic_j;S) \subseteq T(c_i;S)$ and $t_\gamma \in Z(c_ic_j;S)$ implies that there does not exist a $p \in PS(c_ic_j)$ such that $p \in t_\gamma$. Then, we introduce the equivalence relation $\mathbb{C}\,(c_ic_j;S)$ defined on $T(c_i;S) - Z(c_ic_j;S)$, according to which $t_\varphi\,\mathbb{C}\,(c_ic_j;S)t_\psi$ iff there exists a $p \in PS(c_ic_j)$ such that $p \in t_\varphi \cap t_\psi$.

Definition 6. A clock of a physical system S is a physical system formed by $PS(c_ic_j)$, where $c_i, c_j \in S$, and c_j is an evolving particle, and with the property represented by the operator $\Omega(c_ic_j;S)$ such that: (i) $\Omega(c_ic_j;S)$ by acting on $T(c_i;S)$ produces a set $\tilde{T}(c_ic_j;S)$ for which $t_\alpha \in \tilde{T}(c_ic_j;S) \Leftrightarrow t_\alpha \in W(c_ic_j;S)$, and $W(c_ic_j;S) \equiv (T(c_i;S) - Z(c_ic_j;S))/\mathbb{C}\,(c_ic_j;S)$ and (ii) $\tilde{T}(c_ic_j;S)$ is an ordered set such that for $t_\delta, t_\epsilon \in \tilde{T}(c_ic_j;S)$ one has $t_\delta < t_\epsilon$ iff $s_f(x) < s_f(x')$, where $s_f(x)$ and $s_f(x')$ are the first states of $p(c_ic_j|x)$ and $p(c_ic_j|x')$, respectively, and $p(c_ic_j|x) \in t_d \cap PS(c_ic_j)$ and $p(c_ic_j|x') \in t_e \cap PS(c_ic_j)$, with $t_d \in t_\delta$ and $t_e \in t_\epsilon$.

Concerning the condition (ii) of the above definition, let us recall that if $s_f(x)$ and $s_f(x')$ are the first states of $p(c_ic_j|x)$ and $p(c_ic_j|x')$, respectively, then there exist two states of c_j which are precisely these two states. Moreover, because c_j is an evolving particle we must have either $s_f(x) < s_f(x')$ or $s_f(x') < s_f(x)$.

Definition 7. Given a physical system S and two events $e \equiv e(c_m;S)$ and $e' \equiv e(c_n;S)$ with $c_m, c_n \in S$, the time-set-interval between these events with respect to a clock $\Omega(c_ic_j;S)$ is the subset $T_1(c_ic_j;S;ee')$ of $\Omega(c_ic_j;S)T(c_i;S)$, which has first and last element, let them be t_δ and t_ϵ respectively, and there exist two indicators $p(c_ic_m|x)$ and $p(c_ic_n|y)$ such that $p(c_ic_m|x) \in t_d \cap e(c_m;S)$ and $p(c_ic_n|y) \in t_e \cap e(c_n;S)$, where $t_d \in t_\delta$ and $t_e \in t_\epsilon$, and for all $t_\alpha < t_\delta$ and $t_\varphi > t_\epsilon$ one has for every $t_a \in t_\alpha$ and $t_f \in t_\varphi$ that $t_a \cap e(c_m;S) = t_f \cap e(c_n;S) = \emptyset$. Moreover, we define the time interval between $e(c_m;S)$ and $e(c_n;S)$ with respect to the clock $\Omega(c_ic_j;S)$ and to a given measure M, as the measure $\mathscr{M}$ of the set $T(c_ic_j;S;ee') - \{t_\epsilon\}$. If we use cardinality as a measure, the corresponding time interval will be given by the cardinality of the set $T(c_ic_j;S;ee') - \{t_\epsilon\}$.

According to the above definition, given a physical system S, two events $e(c_m;S)$ and $e(c_n;S)$, where $c_m, c_n \in S$, and a clock $\Omega(c_ic_j;S)$, the time-set-interval between $e(c_m;S)$ and $e(c_n;S)$ with respect to $\Omega(c_ic_j;S)$ might not exist, since a time-set-interval must have a first and a last element, which is not always the case for infinite sets. However, all the finite time-set-intervals are trivial cases of doubly-well-ordered sets, which not only have first and last elements but each subset of it has a first and a last element. It may also occur that for two events $e(c_m;S)$ and $e(c_n;S)$, the physical system S concerned be such that for one or two of the events there does not exist a $t_\delta \in \Omega(c_ic_j;S) \times T(c_i;S)$ for which there exists at least one $t_d \in t_\delta$ fulfilling $t_d \cap e(c_k;S) \neq \emptyset$, where k is equal to m or n. In such a case we will say that the events $e(c_m;S)$ and $e(c_n;S)$ are *disconnected* with respect to the clock associated with $\Omega(c_ic_j;S)$.

Given a time-set $T(c_i;S)$, a clock associated with $\Omega(c_ic_j;S)$ and two events $e(c_m;S)$ and $e(c_n;S)$, one may interpret the measurement of the time interval between $e(c_m; S)$ and $e(c_n; S)$ with respect to $\Omega(c_i c_j; S)$ and to the measure m (e.g. the cardinality), as an operation which first may erase some elements of $T(c_i; S)$ yielding $T(c_i; S) - Z(c_i c_j; S)$ and produces a partition of $T(c_i; S) - Z(c_i c_j; S)$, then orders the resulting quotient set, and finally counts the equivalence classes of instants comprised between the two equivalence classes of instants related in the way expressed in Definition 6 to $e(c_m; S)$ and $e(c_n; S)$, respectively. Therefore, the measurement of time intervals depends in a nontrivial way on the clock with respect to which the time interval is defined and on the physical system considered. This provides a very flexible schema for the measurement of time intervals.

3. *Concluding Remarks*

(i) We have developed here a relational theory of time, in the sense that the disappearance of the basic physical constituents of our world would entail the disappearance of time. But time and its relatives on the one hand, and particles on the other hand, are on the same footing since both are based on the base-set of all physical systems, which is formed only by pre-particles.

(ii) Though we have adopted a model of the physical world with an infinite (although denumerable) number of pre-particles, which yields a set of particles with the power of the continuum, one can cover the case of a physical world with a finite number of particles by simply assuming that the base-set B of pre-particles is finite. In this case $\mathscr{P}(B)$ and thus the set of all possible particles will be finite. On the other hand, $\mathscr{P}(B)$ becomes a finite set in which $\subset$ is not connected, and thus can be represented by a graph. The same applies to any particle.

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